List of Projects

Update on 27/06/2023

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1. Turbulent Lagrangian dynamics

proposed by Alessandra

A small data set of Lagrangian trajectroies of tracers and one family of inertial particles in a turbulent flow are given. The dataset contains for each particle: [position, particle velocity, fluid velocity at particle position, velocity gradients] vs [time]

- *Project 1.* Measure PDF of energy dissipation and enstrophy along trajectories; then thresholding energy dissipation, measure the mean first exit-time, as the mean time lag for a tracer starting from below the threshold to achieve the threshold limit. Repeat for enstrophy, compare.
- *Project 2.* Solve Jeffery equation for rotation of an axysimmetric particle in the given turbulent flow. First in a pure simple shear flow frozen in time; then in the flow with all velocity gradients components different from zero, but frozen in time; then in the fully turbulent flow. Characterise statistics of orientation vector.

2. Statistical analysis of turbulent signals

proposed by Laurent

The project will analyze various signals for their hallmark statistical properties (cascades,intermittency,correlations,etc) and propose various levels of stochastic modeling. Sets of data will include hot wire measurements and highly resolved 3d numerical simulations.

3. Spontaneous Stochasticity in Digital Turbulence Models

proposed by Alexei

The project is to observe a development of stochasticity through a multi-scale cascade in discrete models. These models mimic turbulent motion with a discrete number of states. In the simplest version, there are two discrete states, 0 (laminar) and 1 (turbulent), interacting with adjacent scales at discrete turn-over times. This kind of system demonstrates a very rich variety of behaviors,
including finite-time singularities and development of large-scale stochasticity from small-scale noise. In this project, we propose the numerical analysis of digital turbulence models for different forms of interactions and, possibly, its relation to the evolution of renormalization group operators.

4. Numerical investigations of 2D Mikado flows

proposed by Franco

After the pioneering works of Alexander Shnirelman '97 and Camillo De Lellis and LAszlo Szekelyhidi '09, several works produced unexpected phenomena of weak solutions of Euler, Navier-Stokes and other equations, usually going under the name of convex integration theory. A key ingredient is the introduction of small-scale structures suitable for exploiting such phenomena, like for instance the relatively recent Mikado flows. Purpose of this working group is to get introduced to this subject, compare with more classical vortex or eddy structures known in the literature and understand the potential role of them in Turbulence theory.

5. Turbulence meets the pole, from beta-plane to gamma-plane turbulence

proposed by Benjamin

This project will generalise the concept of beta-plane turbulence, relevant to mid-latitude atmospheric and oceanic turbulent flows in thin atmospheres, to gamma-plane turbulence, relevant for their dynamics close to the pole. Using two-dimensional pseudo-spectral numerical simulations (using an open-source code ready to go), we will explore how turbulence behaves as it gets closer to the pole, with and without external forcing. Can we reproduce the vortex crystals observed at the poles of Jupiter?

6. Super-attractors in 3D rotating or stratified cavities

proposed by Benjamin

As an example of a linear cascade, we will consider the propagation of small wavelength inertial or internal waves beams in a closed container. For some geometries and frequencies, all beams can collapse on a critical surface due to focusing reflections on boundaries. We recently showed that super-attractors exist: they are characterised by two focusing directions leading to the collapse of the 3D volume onto a 1D line. So far, we only have found one example of such geometries. Can we find other examples and perhaps derive general conditions for their emergence?

7. Watermarks of a turbulent cascade in single-particle Lagrangian statistics

proposed by Jérémie

Our understanding of turbulence is largely based on the observation that kinetic energy dissipation persists even as viscosity approaches zero (infinite Reynolds number). This feature breaks the time reversibility of the limiting turbulent velocity, and manifests itself through Kolmogorov's 4/5 law, resulting in a negative skewness of velocity increments. In Eulerian statistics, this asymmetry serves as a distinct indication of the turbulent cascade. However, detecting it within a Lagrangian framework is significantly more challenging. This might be due to the fact that traditionally studied quantities along particle paths, such as velocity increments, remain invariant under time-reversal symmetry, making their connection with the irreversible nature of turbulence rather questionable. Some studies have explored alternative asymmetric measures, revealing, for example, that fluid elements undergo time-irreversible fluctuations in their kinetic energy. Nevertheless, a complete understanding of how this relates to the dissipative anomaly, intermittency or other broken symmetries remains to be established. The objective of this project is to analyse Lagrangian datasets obtained from direct numerical simulations at $R_\lambda = 730$, with the following goals:

1. Reproduce established measurements of classical turbulent quantities, including single-time distributions of velocities, gradients, Lagrangian structure functions, energy increments, and more.
2. Conduct a systematic statistical analysis of Lagrangian signals, aiming to identify new insights.
3. Propose and validate alternative observables that can serve as clear indicators of time irreversibility.

8. Predictability of point-vortex systems

proposed by Christophe & Simon

This project intends to analyze the sensitivity to small thermal noise of prototype initial-value problems arising in the 2d fluids, which are candidate set-ups for the spontaneous stochasticity. Spontaneous stochasticity is a mechanism whereby the presence of spatial singularity allow for infinite amplification of thermal noise in finite time, leading to formally deterministic trajectories becoming de facto non-causal. The project proposes to use point-vortex approximations, or generalized versions thereof, to investigate various candidate set-ups of spontaneous stochasticity. One such set-up is the rebounce of a 2d vortex dipole on a smooth wall [Orlandi, '90], which in the presence of small viscosity, is known to lead to formation and detachment of boundary layer. We will investigate whether such behaviour can be retrieved from an inviscid model with or withour suitably tuned stochastic component.

9. Dual cascades in point-vortex sytems

proposed by Sergey & Simon

Conservation of vorticity along fluid paths in the 2D Euler equation allows to discretise 2D fluid motions, including 2D turbulence, by a "gas" of point vortices. Such a point vortex model is a nice reduction from the Euler PDE to a simpler set (large but finite) of ODEs. As a result, one can apply to this model a thermodynamic description, which was first done by Lars Onsager, who famously predicted existence of negative temperature states describing clustering of like-signed vortices thereby forming large-scale vortices. In fact, emergence of these states is a manifestation of what was later called the inverse energy cascade. The point vortex model is especially important in superfluid turbulence where the vortices are point-like and their circulations are quantised. In addition, the opposite signed quantised vortices can annihilate which provides an energy sink mechanism which enables formation of the Onsager clusters. We will start with introducing the point vortex model and deriving basic predictions using the thermodynamic approach. We will also review the basic theory of the dual cascade in 2D turbulence. Then, we will simulate the motion of a point vortex set numerically using a Python code provided thereby testing the theoretical predictions about the forward and inverse cascades, as well as formation of the Onsager states. Finally, we will analyse a provided dataset obtained by running a much larger vortex ensemble in order to identify the turbulence scaling properties, including the power-law energy spectra.