Lectures in Probability and Statistics
2024-25
Master Maths — Nice

Stochastic Calculus (Rémi Catellier)

The course "Stochastic Calculus and Applications" is designed to provide students with a deep understanding of stochastic processes, with a focus on both theoretical foundations and practical applications. Key topics include Brownian motion, a fundamental stochastic process that models random behavior in various contexts, and continuous-time martingales, which are essential for understanding fair game-like properties in financial mathematics. The course also covers the stochastic integral, a tool used to model systems influenced by random noise, as well as stochastic differential equations (SDEs), which allow the modeling of dynamic systems affected by randomness.

In addition to the theoretical framework, the course explores a variety of real-world applications. For instance, stochastic methods are widely used in machine learning for optimization and in the pricing of derivative products in financial markets. Furthermore, stochastic models play a crucial role in biological systems, such as in population dynamics and the spread of diseases. By bridging theory with applications, this course equips students with the skills to apply stochastic calculus in a broad range of domains.

Numerical Methods in Probability (Sylvain Rubenthaler)

- This course addresses the basic methods used for simulating random variables and implementing Monte-Carlo and Quasi Monte-Carlo methods.
- Simulation of stochastic processes used in mathematical finance, such as Brownian motion and solutions to stochastic differential equations, will be addressed.
- The course will introduce sampling methods in finite dimensions, discretization of diffusion processes, strong and weak errors.

Advanced Stochastics (François Delarue)

This course follows the course on stochastic calculus given by Prof. Catellier. The purpose is to push the analysis further and in particular to address three main questions that artery frequent in probability theory and related modelling:

- Long time behaviour of time homogeneous stochastic differential equations. Here the objective is to find some conditions under which the stochastic differential equation has a unique invariant measure to which the solution converges in long time.
- Stochastic differential equations in very high dimension. In this example, we address an example of a stochastic differential equation in very high dimension in which coordinates interact with one another in a weak manner. We show that there is a limit regime when the number of coordinates tends to infinity. This regime is known as `mean-field'.
- Control theory. We study a class of controlled stochastic differential equations. The purpose is to find a systematic procedure to find the best possible control in order to minimise some energy (or cost).

Statistical learning with and on graphs (Marco Corneli)

After a quick refresher about maximum likelihood inference, with application to linear and logistic regression, the course will focus on graphical models, in particular on directed a-cyclic graphs (DAGs). A chapter about mixture models will follow, illustrating their relation with data clustering and the expectation maximization (EM) algorithm will be presented and discussed in detail. Then, we will attack the topic of generative models for random graphs and their application to the clustering of the nodes of a graph. Variational inference for stochastic block models will be discussed in detail. A last chapter is about graph neural networks and their use for supervised learning tasks at the instance level.

Each topic will be first discussed in detail from a theoretical point of view. Then, its application will be illustrated by means of Python notebooks or R markdowns (the student will become familiar with known machine learning libraries).

Some teaching material will be provided by the teacher. Additional useful references are

- Wasserman, L. All of Statistics: a Concise Course in Statistical Inference, Springer, 2013. [Chapters 9,13,16,17].

- Bishop, Christopher M. Pattern recognition and machine learning, Springer, 2006. [Chapters 3,4,8].

- Bishop, Christopher M., and Hugh Bishop. Deep learning: Foundations and concepts, Springer Nature, 2023. [Chapter 13]

- Daudin, J-J., Franck Picard, and Stéphane Robin. A mixture model for random graphs. Statistics and computing 18.2 (2008): 173-183.

Geometric Statistics (Khazhgali Kozhasov)

Statistical concepts like principal component analysis, (empirical) mean or covariance (matrix) are inherent to data and probability distributions living in linear spaces. Geometric statistics aims at providing tools for analysing data that populate (possibly) non-linear spaces such as manifolds. As the notion of metric is essential for this goal, Riemannian geometry provides a solid ground for the theory. In the course we are going to introduce necessary geometric results, give essentials on probability distributions and then discuss "nonlinear" generalizations of some classical concepts from statistics. The exposition will be accompanied by numerous examples with a view towards applications. Familiarity with calculus on manifolds or basic differential geometry is recommended.

[1] Sampling from a Manifold, P. Daconis, S. Holmes and M. Shahshahani, Advances inModern Statistical Theory and Applications: A Festschrift in honor of Morris L. Eaton, 102–125, 2013

[2] Principal Geodesic Analysis for the Study of Nonlinear Statistics of Shape, P. T. Fletcher, C. Lu, S. M. Pizer and S. Joshi, IEEE transactions on medical imaging, 23(8), 995–1005, 2004

[3] Riemannian Geometric Statistics in Medical Image Analysis, T. Fletcher, X. Pennec and S. Sommer, Elsevier, https://doi.org/10.1016/C2017-0-01561-6, 2020

[4] Les éléments aléatoires de nature quelconque dans un espace distancié, M. Fréchet, Annales de l'institut Henri Poincar´e, 10(4), 215–310, 1948

[5] Introduction to Riemannian Geometry and Geometric Statistics: from basic theory to implementation with Geomstats, N. Guigui, N. Miolane and X. Pennec, Foundations and Trends in Machine Learning, 2023