Numerical methods for pde

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There are many different discretization strategies to handle PDEs: Finite Differences, Finite Element, Finite Volume, Spectral methods… The purpose of this lecture is to present Finite Element and Finite Volume methods for elliptic equations. Finite Volume are intended to mimic the relations obtained by integrating over cells the PDE under consideration, and they reproduce at the discrete level the balance principles that have led to the derivation of the equations. Such methods apply to many different fields: fluid mechanics, heat and mass transfer in heterogeneous media,...

After introducing the main ideas and construction principles of the methods, we review some results of the recent literature, focusing on two important properties of schemes (discrete versions of well-known properties of the continuous equation): coercivity and minimum-maximum principles. Coercivity ensures the stability of the method as well as its convergence under assumptions compatible with real-world applications, whereas minimum-maximum principles are crucial in case of strong anisotropy to obtain physically meaningful approximate solutions.
The outline of the lecture is the following:

- Presentation of the main ideas of the finite volume methods.
- Study in dimension 1:
    - Coercivity, well-posedness and minimum-maximum principles of the discrete solution.
    - Convergence of the discrete solution.
- Extension to dimension 2:
    - Study of the two points flux approximation (classical strategy).
    - Study of a more general method called Discrete Duality Finite Volume.
- Numerical simulations using scilab.
- Presentation of the main ideas of the finite element methods.
- Numerical simulations using FreeFem++.