Mathematical tools for pde

Ivan Moyano

Introduction to nonlinear problems, a priori estimation, compactness methods, hyperbolic problems.
This lecture is an introduction to mathematical theory of PDEs, such as the Laplace equation, the heat equation, the transport equation and the wave equation.

Laplace and Poisson equation: $-\Delta u = f$,
Heat equation $\partial t\ u - \Delta u = f$,
Transport equation $\partial t\ u+c(x)\nabla u = f$ and continuity equation $\partial t\ u+div(c(x)u) = f$
Wave equation $\partial t2\ u - c2\ \Delta u = f$,

where u = u(x) or u(t, x) is the unknown scalar or vectorial function, f (x) or f (t, x) is a stated function , f = 0 at the beginning, c is the fixed celerity. For these important examples, basic mathematical concepts and tools will be introduced and used: separation of variables, functional spaces, traces for boundary conditions, fixed point, energy method, Fourier analysis, weak formulation, strong and weak solution, characteristics, cone of dependence, . . .

[B] Brezis, Haim. Functional analysis, Sobolev spaces and partial differential equations, 2011. xiv+599 pp.
Evans, Lawrence, Partial Differential Equations, AMS