M2 Introductory Course Offering for the Algebra and Geometry Block

20 December 2023

1 Introduction

The aim of our M2 course proposal is to illustrate the ways in which topological, geometric, analytic and algebraic approaches complement each other and naturally shed light on the study of varieties, analytical and algebraic approaches naturally complement and illuminate each other in the study of varieties. In particular, the aim of this unit is to provide an introduction to the various aspects (topological, geometric, analytic, algebraic) of varieties, analytic, algebraic) of complex varieties.

An large part will be devoted to the study of complex curves and therefore of Riemann surfaces, a rich and central notion in many fields of mathematics. Riemann surfaces, a rich and central concept in many very active areas of research. Additionally, we will study the different geometries that give a topological surface the structure of a Riemann surface, as well as the invariants of Riemann surfaces. topological surface, as well as the associated analytic invariants. We will also study the cohomology with applications such as the Riemann-Roch theorem.

Key words: complex varieties, algebraic curves, Riemann surfaces, bundles, Riemann-Roch

2 Introductory Course: Complex Geometry

Teacher: S. Dumitrescu

Complex varieties (smooth or singular) are very rich geometric objects that can be studied from both analytical and algebraic points of view. We will begin by introducing complex varieties of arbitrary dimension and the basic tools needed to study them. In the second part of the course we look in more detail at the case of algebraic curves (or Riemann surfaces).

2.1 Part 1: Complex Varieties

— Holomorphic functions in several variables
— Analytic sets in $\mathbb{C}^n$
— Smooth complex varieties, projective varieties, (holomorphic) vector fibres
— differential forms and forms of type (p, q), Dolbeault and de Rham cohomology - Riemann-Roch theorem on a compact Riemann surface

2.2 Part 2: Riemann Surfaces

— Definition. Complex varieties of dimension 1. Fundamental examples: complex plane, upper half-plane, Riemann sphere. Elliptic curves.
— Algebraic curves and associated Riemann surfaces. Riemann surface associated with an analytic function. Example of log.
— Riemann surfaces obtained as quotients by a discrete group of automorphisms. Molecular group. Modular surface.
— Riemannian metrics on surfaces. Conformal transformations. Hyperbolic geometry.
— Isothermal coordinate theorem. Uniformity theorem. Consequences.

References

[Fis76] Gerd Fischer. *Complex analytic geometry*. Springer-Verlag, Berlin, 1976. Lecture Notes in Mathematics, Vol. 538.

[For91] Otto Forster. *Lectures on Riemann surfaces*, volume 81 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, 1991. Translated from the 1977 German original by Bruce Gilligan, Reprint of the 1981 English translation.

[KK83] Ludger Kaup and Burchard Kaup. *Holomorphic functions of several variables*, volume 3 of *de Gruyter Studies in Mathematics*. Walter de Gruyter & Co., Berlin, 1983. An introduction to the fundamental theory, With the assistance of Gottfried Barthel, Translated from the German by Michael Bridgland.

[Voi02] Claire Voisin. *Théorie de Hodge et géométrie algébrique complexe*, volume 10 of *Cours Spécialisés*. Société Mathématique de France, Paris, 2002.

[SG10] Henri Paul de Saint-Gervais *Uniformisation des surfaces de Riemann* ENS Editions, 2010