Introduction to Several Complex Variables

The modern theory of functions of several complex variables can reasonably be dated to the researches of F. Hartogs and K. Oka in the first decade of the twentieth century.
The so-called Hartogs Phenomenon («Isolated singularity is removable, for any analytic function of several variables»), a fundamental feature that had eluded Weierstrass, reveals a dramatic difference between one-dimensional complex analysis and multidimensional complex analysis.
After important works in France (Cartan's seminar) and in Germany (H. Grauert and R. Remmert) quickly changed the picture of the theory. A number of issues were clarified, in particular that of analytic continuation. Here a major difference is evident from the one-variable theory: while for any open connected set *D* in **C** we can find a function that will nowhere continue analytically over the boundary, that cannot be said for $n > 1$. In fact the *D* of that kind are rather special in nature (a condition called pseudoconvexity).

Ever since the theory of functions of several complex variables has developed in various directions. One of these approaches is to use precise estimates on the Cauchy-Riemann equations in spaces of square integrable functions (a wide assortment of different L^2 norms with different plurisubharmonic functions) and differential forms to obtain quantitative understanding of holomorphic functions theory and analytic geometry on complex manifolds of higher dimensions. Referred to as the "*L*2-theory of the ∂bar-problem", this method originated in the 1960s in the work of Hörmander, Kohn and Andreotti-Vesentini.
Since that time this theory has evolved into a very powerful and flexible tool to construct analytic objects.

Some aspects of the theory of holomorphic (complex analytic) functions, are essentially the same in all dimensions. The multi-dimensional theory reveals striking new phenomena (for power series expansions, integral representations, partial differential equations and geometry, for example).
In this course, we will introduce this theory and will sketch some of these phenomena.
There exists a large scope of the interaction between complex analysis and other parts of mathematics, including geometry, partial differential equations, probability, functional analysis, number theory, algebra, and mathematical physics.
One of the goals of this course will be to glimpse some connections between complex analysis and analytic geometry and functional analysis.

Holomorphic Functions in Several Variables
Holomorphic Convexity and Pseudoconvexity
CR Functions
The $\bar{\partial}$-problem
Integral Kernels

Complex Analytic Varieties

References:

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R. Narasimhan, Several Complex Variables, University of Chicago Press, Chicago, 1971.14.

R. M. Range, Holomorphic Functions and Integral Representations in Several Complex Variables,Springer-Verlag, New York, 1986; 2nd. corrected printing, 1998.

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Prerequisites: The following topics will be assumed known.

1. Basic theory of holomorphic functions of one complex variable.

2. Real Analysis: Basic facts about measure and integration in Euclidean spaces. Differential calculus of several real variables.

3. Basic facts about Banach spaces and Hilbert spaces.

4. Basic Algebra (vector spaces, groups,rings etc.).