L.J.A.D.
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M2 course proposal
Differential Topology and Geometry

The course aims to give an introduction to differential geometry and topology, emphasizing a differential approach.

Part 1 *Differential calculus.* In this first part, we want to introduce the basic tools of differential geometry

1. Manifolds, smooth functions and mappings.
2. Partitions of unity, embedding of manifolds.
3. Differential forms, the exterior derivative, the Poincaré Lemma and the Stokes theorem.
4. Vector fields, flow, Lie derivative. Frobenius theorem and integrable planes.

Part 2 *Differential topology.* In the second part we give an introduction to algebraic topology from the point of view of De Rham cohomology

1. De Rham cohomogy, definitions and computations: the Mayer–Vietoris sequence, with an introduction to Poincaré duality.
2. (If time permits) Sard Theorem and transversality. The degree theory.

Part 3 *Vector bundles and connections.* In a third part, we explain the notion of a vector bundle (over a topological space) and why these objects gives topological invariants.

1. Definitions of vector bundles: algebraic constructions.
2. What problem do connections solve on a vector bundle? definition of a Koszul connection.
3. Parrallel transport, flatness and curvature

4. (If time permits) Characteristic classes and the Chern-Weyl theory and how do vector bundles create cohomology classes.

Part 3 *Complement* We plan to give, through student talks introduction to other subjects:

1. Riemaniann geometry: Levi–Civita connection and spaces of constant curvature.
2. Representations of the fundamental group.
3. Simplicial homology and cohomology.

Below is a list of standard references [3][4][1][2].

References

[1] Sylvestre Gallot, Dominique Hulin, and Jacques Lafontaine, *Riemannian geometry*, third ed., Universitext, Springer-Verlag, Berlin, 2004. MR 2088027

[2] François Labourie, *Lectures on representations of surface groups*, Zurich Lectures in Advanced Mathematics, European Mathematical Society (EMS), Zürich, 2013.

[3] John W. Milnor, *Topology from the differentiable viewpoint*, Princeton Landmarks in Mathematics, Princeton University Press, Princeton, NJ, 1997, Based on notes by David W. Weaver, Revised reprint of the 1965 original. MR 1487640

[4] John W. Milnor and James D. Stasheff, *Characteristic classes*, Annals of Mathematics Studies, vol. No. 76, Princeton University Press, Princeton, NJ; University of Tokyo Press, Tokyo, 1974. MR 440554

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