Advanced Partial Differential Equations Analysis, Hyperbolic conservation laws

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This lecture deals on PDEs where the nonlineraity, the time and the finite speed of propagation of the information play an important role such as the Euler equations for the gas dynamics.

$$\begin{aligned} \partial_t \rho + \operatorname{div}(\rho u) &= 0, \\ \partial \rho u + \operatorname{div}(\rho u \otimes u) + \nabla p(\rho) &= 0, \end{aligned}$$

where ρ is the density, u the velocity and p the pressure.

We start from the linear and nonlinear waves equations, the importance of the dimension $d = 1$, $d > 1$, the Euler system, the firts studies on smooth and non-smooth solutions, the vanishing viscosity method, the shock waves, the entropy.

[D] Dafermos, Constantine M. Hyperbolic conservation laws in continuum physics. 2016. xxxviii+826 pp.

[H] Hörmander, Lars. Lectures on nonlinear hyperbolic differential equations. 1997. viii+289 pp.

[L] Lax, Peter D. Hyperbolic partial differential equations. 2006. viii+217 pp.

[S] Smoller, Joel. Shock waves and reaction-diffusion equations 1994, xxiv+632 pp.